Written exam at the Department of Economics
Summer 2020
Monetary Policy
Suggested Answers

PROBLEM A

1) True. Taking as given the level of inflation fluctuations, the welfare loss will be increasing with a higher elasticity of substitution between different individual goods, which is negatively correlated with the steady-state mark-up. Thus, a higher elasticity of substitution makes it easier for households to substitute between different individual goods and, in turn, to increase their consumption of those goods that are relatively cheap at the expense of a reduced consumption of relatively expensive goods. This leads to a higher welfare loss, since dispersion of consumption between individual goods is a source of inefficiency: while all goods enter the utility function in a symmetric way, there is decreasing marginal utility of consumption, as well as decreasing returns to scale in production, which makes it efficient to ensure that all individual goods are produced and consumed in the same quantity. Thus, the statement is true.

2) False. The UIP condition implies that:

$$\mathrm{E}_t s_{t+1} - s_t = i_t - i_t^*,$$

while the real exchange rate is defined as:

$$q_t \equiv s_t - p_t + p_t^*.$$

Take the difference of the real exchange rate, and take expectations:

$$\begin{aligned} \mathrm{E}_t q_{t+1} - q_t &= \mathrm{E}_t s_{t+1} - s_t - (\mathrm{E}_t p_{t+1} - p_t) + \left(\mathrm{E}_t p_{t+1}^* - p_t^*\right) \Leftrightarrow \\ \mathrm{E}_t s_{t+1} - s_t &= \mathrm{E}_t q_{t+1} - q_t + (\mathrm{E}_t p_{t+1} - p_t) - \left(\mathrm{E}_t p_{t+1}^* - p_t^*\right), \end{aligned}$$

which we can rewrite using the UIP as:

$$i_t - i_t^* = \mathrm{E}_t q_{t+1} - q_t + (\mathrm{E}_t p_{t+1} - p_t) - \left(\mathrm{E}_t p_{t+1}^* - p_t^*\right).$$

Now apply the hints given in the exercise, which state that $i_t^* = 0$ and $\mathrm{E}_t p_{t+1}^* - p_t^* = 0$, to obtain:

$$\begin{aligned} i_t &= \mathrm{E}_t q_{t+1} - q_t + \mathrm{E}_t p_{t+1} - p_t \Leftrightarrow \\ i_t &= \mathrm{E}_t q_{t+1} - q_t + \mathrm{E}_t \pi_{t+1}. \end{aligned}$$

This shows that the domestic nominal interest rate is given by the *expected* change in the real exchange rate plus *expected* domestic inflation. The statement is therefore false, since it posed that the nominal interest rate was given by the *actual* values of these variables (which may turn out to be true *ex post*, but is not generally true).

3) False. Intertemporal substitution of consumption, or the so-called "direct effect" of monetary policy, is by far the main transmission mechanism in representative-agent New Keynesian ("RANK") models. However, as emphasized in the HANK literature studied during the course, these effects play only a small role in models with heterogeneous agents and uninsurable idiosyncratic risk. In these models, a large fraction of household behave in a hand-to-mouth manner, implying that their consumption responds little to interest rate changes, but strongly to changes in labor income. In HANK models, therefore, monetary policy mainly operates through "indirect effects" such as the general equilibrium increase in labor demand, changes in asset prices, etc. From a quantitative perspective, the "direct effect" through intertemporal substitution accounts for more than 90 percent of the impact of monetary policy on consumption in RANK models, but only around 20 percent in HANK models.

PROBLEM B

1) Equation B.1 is the New-Keynesian Phillips curve (NKPC). For given inflation expectations, it implies a positive relationship between inflation and the output gap: An increase in the output gap raises the marginal cost faced by firms. This leads to higher prices and hence inflation. Moreover, the NKPC also implies that current inflation increases if agents expect inflation to increase in the future: Since firms are subject to sticky prices, they know that they may not be able to change their price for some periods into the future. Hence, if they expect high inflation, it will be optimal for them to raise their price already today, if allowed to. The NKPC is derived from the pricing decisions of firms, taking into account

the labor supply decision of households as well as the production function, which both affect the marginal cost faced by firms.

Equation B.2 is the dynamic IS curve (DIS). It is derived by combining the household's Euler equation for consumption with the goods market clearing condition. It implies a relationship between the current and future output gap and the expected real interest rate (in deviations from its natural or steady state level): When the real interest rate is (expected to be) high, saving for the future becomes more attractive, and current consumption therefore less attractive. As a result, economic activity is moved from the present to the future, so the current output gap drops (becomes negative), while the future output gap increases.

Equation B.3 is an interest rate rule specifying how the central bank sets the interest rate in response to movements in inflation and the output gap. It is often referred to as a *Taylor rule*. The assumption that $\phi_\pi > 1$ ensures that the central bank raises the nominal interest rate more than one-for-one in response to an increase in inflation, thereby raising also the real interest rate. In turn, this depresses current economic activity, and thereby brings inflation back down. This assumption, which is often referred to as the *Taylor principle*, is necessary to ensure a unique determinate equilibrium.

2) We make the following conjecture:

$$\begin{aligned}\widetilde{y}_t &= b_1 v_t, \\ \pi_t &= c_1 v_t.\end{aligned}$$

Note first that this has the following implications for the expectations of these two variables:

$$\begin{aligned}\mathrm{E}_t \pi_{t+1} &= c_1 \mathrm{E}_t v_{t+1} \Leftrightarrow \\ \mathrm{E}_t \pi_{t+1} &= c_1 \rho_v v_t,\end{aligned}$$

$$\mathrm{E}_t \tilde{y}_{t+1} = b_1 \rho_v v_t.$$

Now insert the guesses and these expectations into the original system of equations. First into the NKPC:

$$\begin{aligned}\pi_t &= \beta \mathrm{E}_t \left\{\pi_{t+1}\right\} + \kappa \widetilde{y}_t \Leftrightarrow \\ c_1 v_t &= \beta c_1 \rho_v v_t + \kappa b_1 v_t \Leftrightarrow \\ c_1 &= \frac{\kappa}{1 - \beta \rho_v} b_1.\end{aligned}$$

Now operate on the DIS, first using the Taylor rule:

$$\begin{aligned}
\widetilde{y}_t &= -\frac{1}{\sigma}\left(i_t - \mathrm{E}_t\left\{\pi_{t+1}\right\} - \rho\right) + \mathrm{E}_t\left\{\widetilde{y}_{t+1}\right\} \Leftrightarrow \\
\widetilde{y}_t &= -\frac{1}{\sigma}\left(\left[\rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + v_t\right] - \mathrm{E}_t\left\{\pi_{t+1}\right\} - \rho\right) + \mathrm{E}_t\left\{\widetilde{y}_{t+1}\right\} \Leftrightarrow \\
\widetilde{y}_t &= -\frac{1}{\sigma}\left(\phi_\pi \pi_t + \phi_y \widetilde{y}_t + v_t - \mathrm{E}_t\left\{\pi_{t+1}\right\}\right) + \mathrm{E}_t\left\{\widetilde{y}_{t+1}\right\}.
\end{aligned}$$

Insert from the conjecture:

$$\begin{aligned}
\widetilde{y}_t &= -\frac{1}{\sigma}\left(\phi_\pi \pi_t + \phi_y \widetilde{y}_t + v_t - \mathrm{E}_t\left\{\pi_{t+1}\right\}\right) + \mathrm{E}_t\left\{\widetilde{y}_{t+1}\right\} \Leftrightarrow \\
b_1 v_t &= -\frac{1}{\sigma}\left(\phi_\pi c_1 v_t + \phi_y b_1 v_t + v_t - c_1 \rho_v v_t\right) + b_1 \rho_v v_t \Leftrightarrow \\
b_1 &= -\frac{1}{\sigma}\left(\phi_\pi c_1 + \phi_y b_1 + 1 - c_1 \rho_v\right) + b_1 \rho_v \Leftrightarrow \\
b_1\left(1 + \frac{\phi_y}{\sigma} - \rho_v\right) &= c_1\left(\frac{\rho_v - \phi_\pi}{\sigma}\right) - \frac{1}{\sigma} \Leftrightarrow \\
b_1 &= \left(\frac{\rho_v - \phi_\pi}{\phi_y + (1-\rho_v)\sigma}\right) c_1 - \frac{1}{\phi_y + (1-\rho_v)\sigma}.
\end{aligned}$$

Now insert into the expression derived from the NKPC:

$$\begin{aligned}
c_1 &= \frac{\kappa}{1-\beta\rho_v} b_1 \Leftrightarrow \\
c_1 &= \frac{\kappa}{1-\beta\rho_v}\left(\frac{\rho_v - \phi_\pi}{\phi_y + (1-\rho_v)\sigma}\right) c_1 - \frac{\kappa}{1-\beta\rho_v}\frac{1}{\phi_y + (1-\rho_v)\sigma} \Leftrightarrow
\end{aligned}$$

$$\begin{aligned}
c_1\left[1 - \frac{\kappa}{1-\beta\rho_v}\left(\frac{\rho_v - \phi_\pi}{\phi_y + (1-\rho_v)\sigma}\right)\right] &= -\frac{\kappa}{1-\beta\rho_v}\frac{1}{\phi_y + (1-\rho_v)\sigma} \Leftrightarrow \\
c_1\left(1 + \frac{\kappa(\phi_\pi - \rho_v)}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right]}\right) &= -\frac{\kappa}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right]} \Leftrightarrow \\
c_1 \frac{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right]} &= -\frac{\kappa}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right]} \Leftrightarrow \\
c_1 &= -\frac{\kappa}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)}.
\end{aligned}$$

Then plug back in to solve for b_1:

$$
\begin{aligned}
c_1 &= \frac{\kappa}{1-\beta\rho_v} b_1 \Leftrightarrow \\
b_1 &= -\frac{1-\beta\rho_v}{\kappa} \frac{\kappa}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)} \Leftrightarrow \\
b_1 &= -\frac{1-\beta\rho_v}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)}.
\end{aligned}
$$

So we have the following solutions:

$$
\tilde{y}_t = -\frac{1-\beta\rho_v}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)} v_t, \tag{1}
$$

$$
\pi_t = -\frac{\kappa}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)} v_t. \tag{2}
$$

It is possible to determine the sign of these coefficients: Both the numerators and the (common) denominator is positive, since all parameters are non-negative and since ρ_v and β are both smaller than 1, while $\phi_\pi > 1$. The minus in front of the expressions thus determines a negative response of both variables to an increase in v_t, i.e. a contractionary monetary policy shock. An increase in the nominal interest rate raises also the real rate, making saving more attractive, so consumption and output drops. Firms respond to the drop in demand by reducing their prices if they can.

3) Inserting the the solutions for $\tilde{y}_t$ and π_t in the interest rate rule, we obtain:

$$
\begin{aligned}
i_t &= \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \Leftrightarrow \\
i_t &= \rho - \frac{\kappa\phi_\pi}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)} v_t \\
&\quad - \frac{\phi_y(1-\beta\rho_v)}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)} v_t + v_t,
\end{aligned}
$$

which can be simplified to yield:

$$
\begin{aligned}
i_t &= \rho + \left[1 - \frac{\kappa\phi_\pi + \phi_y(1-\beta\rho_v)}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)}\right] v_t \Leftrightarrow \\
i_t &= \rho + \frac{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v) - \kappa\phi_\pi - \phi_y(1-\beta\rho_v)}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)} v_t \Leftrightarrow \\
i_t &= \rho + \frac{(1-\beta\rho_v)(1-\rho_v)\sigma - \kappa\rho_v}{(1-\beta\rho_v)\left[\phi_y + (1-\rho_v)\sigma\right] + \kappa(\phi_\pi - \rho_v)} v_t.
\end{aligned}
$$

It is not possible to determine the sign of the response of i_t. While the denominator is positive, as argued above, we cannot say whether the numerator is positive or negative. On one hand, the direct impact of the monetary policy shock is to drive up the nominal interest rate. On the other hand, this reduces inflation and the output gap, as we saw in the previous question. According to the Taylor rule, this induces the central bank to reduce the nominal rate. These are the different forces at play. Note that this question can also be answered even if the student was unable to arrive at the solutions $\widetilde{y}_t$ and π_t in question 2. In the textbook by Galí (2015), it is argued that the nominal interest rate will increase in response to a contractionary monetary policy shock unless the monetary policy shock itself is very persistent. Since we have not assumed anything about the persistence of the shock, we cannot give a definitive answer.

4) As prices become more flexible, κ increases. Thus, the negative force induced on the output gap from a monetary policy shock decreases. This explains why, mechanically, the interest rate policy instrument is less reactive. Intuitively, more flexible prices imply that the policy maker needs to stimulate the output gap proportionally less, so as to control inflation, so that the response of the policy instrument to the non-systematic part of the policy rule is more muted.

PROBLEM C

1) We conjecture that the solution is of the form:

$$\begin{aligned} x_t &= b_1 u_t + b_2 \varepsilon_t, \\ \pi_t &= c_1 u_t + c_2 \varepsilon_t, \end{aligned}$$

where we then need to find the parameters b_1, b_2, c_1, and c_2. Since both shocks are i.i.d. mean zero, the conjectures imply:

$$\begin{aligned} \mathrm{E}_t x_{t+1} &= 0, \\ \mathrm{E}_t \pi_{t+1} &= 0. \end{aligned}$$

Insert the conjectures and their expectations into C.1:

$$c_1 u_t + c_2 \varepsilon_t = \kappa \left(b_1 u_t + b_2 \varepsilon_t \right) + u_t. \tag{3}$$

Insert the conjectures and their expectations, and the interest rate rule, into C.2:

$$b_1 u_t + b_2 \varepsilon_t = -\frac{1}{\sigma}\left(\phi_\pi \left(c_1 u_t + c_2 \varepsilon_t\right)\right) + \varepsilon_t. \tag{4}$$

Since (3) and (4) must hold for all u_t and ε_t, we can combine them to obtain:

$$\begin{aligned}
c_1 &= \kappa b_1 + 1, \\
c_2 &= \kappa b_2, \\
b_1 &= -\frac{\phi_\pi c_1}{\sigma}, \\
b_2 &= -\frac{\phi_\pi c_2}{\sigma} + 1.
\end{aligned}$$

We can combine the second and fourth equation to obtain:

$$\begin{aligned}
b_2 &= -\frac{\phi_\pi \kappa b_2}{\sigma} + 1 \Leftrightarrow \\
b_2\left(1 + \frac{\phi_\pi \kappa}{\sigma}\right) &= 1 \Leftrightarrow \\
b_2 &= \frac{\sigma}{\sigma + \phi_\pi \kappa}, \qquad (5)
\end{aligned}$$

and then:

$$c_2 = \frac{\kappa\sigma}{\sigma + \phi_\pi \kappa}. \tag{6}$$

Likewise, the first and third equation yield:

$$\begin{aligned}
c_1 &= -\frac{\kappa \phi_\pi c_1}{\sigma} + 1 \Leftrightarrow \\
c_1 &= \frac{\sigma}{\sigma + \phi_\pi \kappa}, \qquad (7)
\end{aligned}$$

and:

$$b_1 = -\frac{\phi_\pi}{\sigma + \phi_\pi \kappa}. \tag{8}$$

We can then summarize the solutions as follows:

$$x_t = -\frac{\phi_\pi}{\sigma + \phi_\pi \kappa} u_t + \frac{\sigma}{\sigma + \phi_\pi \kappa} \varepsilon_t, \tag{9}$$

$$\pi_t = \frac{\sigma}{\sigma + \phi_\pi \kappa} u_t + \frac{\kappa\sigma}{\sigma + \phi_\pi \kappa} \varepsilon_t. \tag{10}$$

Since u_t and ε_t are both i.i.d. and mutually uncorrelated, it follows that the variances of the output gap and inflation are given by:

$$\sigma_x^2 = \left(\frac{\phi_\pi}{\sigma + \phi_\pi \kappa}\right)^2 \sigma_u^2 + \left(\frac{\sigma}{\sigma + \phi_\pi \kappa}\right)^2 \sigma_\varepsilon^2,$$

$$\sigma_\pi^2 = \left(\frac{\sigma}{\sigma + \phi_\pi \kappa}\right)^2 \sigma_u^2 + \left(\frac{\kappa\sigma}{\sigma + \phi_\pi \kappa}\right)^2 \sigma_\varepsilon^2.$$

2) We begin by inserting the variances in the social loss function:

$$\mathbb{L} = \eta \sigma_x^2 + \sigma_\pi^2 \Leftrightarrow$$

$$\mathbb{L} = \eta \left[\left(\frac{\phi_\pi}{\sigma + \phi_\pi \kappa}\right)^2 \sigma_u^2 + \left(\frac{\sigma}{\sigma + \phi_\pi \kappa}\right)^2 \sigma_\varepsilon^2\right] + \left(\frac{\sigma}{\sigma + \phi_\pi \kappa}\right)^2 \sigma_u^2 + \left(\frac{\kappa\sigma}{\sigma + \phi_\pi \kappa}\right)^2 \sigma_\varepsilon^2 \Leftrightarrow$$

We then differentiate this expression with respect to ϕ_π:

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial \phi_\pi} &= 2\eta \left(\frac{\phi_\pi}{\sigma + \phi_\pi \kappa}\right) \sigma_u^2 \left(\frac{(\sigma + \phi_\pi \kappa) - \phi_\pi \kappa}{(\sigma + \phi_\pi \kappa)^2}\right) + 2\eta \left(\frac{\sigma}{\sigma + \phi_\pi \kappa}\right) \sigma_\varepsilon^2 \left(\frac{-\kappa\sigma}{(\sigma + \phi_\pi \kappa)^2}\right) + \\ &\quad + 2 \left(\frac{\sigma}{\sigma + \phi_\pi \kappa}\right) \sigma_u^2 \left(\frac{-\kappa\sigma}{(\sigma + \phi_\pi \kappa)^2}\right) + 2 \left(\frac{\kappa\sigma}{\sigma + \phi_\pi \kappa}\right) \sigma_\varepsilon^2 \left(\frac{-\sigma\kappa^2}{(\sigma + \phi_\pi \kappa)^2}\right). \end{aligned}$$

Set this to zero and solve for ϕ_π:

$$\frac{\partial \mathbb{L}}{\partial \phi_\pi} = 0 \Leftrightarrow$$

$$\eta \phi_\pi \sigma_u^2 \left(\frac{\sigma}{(\sigma + \phi_\pi \kappa)^3}\right) + \eta \sigma_\varepsilon^2 \left(\frac{-\kappa\sigma^2}{(\sigma + \phi_\pi \kappa)^3}\right) + \sigma_u^2 \left(\frac{-\kappa\sigma^2}{(\sigma + \phi_\pi \kappa)^3}\right) + \sigma_\varepsilon^2 \left(\frac{-\sigma^2\kappa^3}{(\sigma + \phi_\pi \kappa)^3}\right) = 0 \Leftrightarrow$$

$$\eta \phi_\pi \sigma_u^2 \left(\frac{\sigma}{(\sigma + \phi_\pi \kappa)^3}\right) - \eta\kappa\sigma\sigma_\varepsilon^2 \left(\frac{\sigma}{(\sigma + \phi_\pi \kappa)^3}\right) - \kappa\sigma\sigma_u^2 \left(\frac{\sigma}{(\sigma + \phi_\pi \kappa)^3}\right) - \sigma\kappa^3\sigma_\varepsilon^2 \left(\frac{\sigma}{(\sigma + \phi_\pi \kappa)^3}\right) = 0 \Leftrightarrow$$

$$\begin{aligned} \eta \phi_\pi \sigma_u^2 &= \eta\kappa\sigma\sigma_\varepsilon^2 + \kappa\sigma\sigma_u^2 + \sigma\kappa^3\sigma_\varepsilon^2 \Leftrightarrow \\ \phi_\pi &= \frac{\kappa\sigma\eta\sigma_\varepsilon^2 + \kappa\sigma\sigma_u^2 + \sigma_\varepsilon^2\sigma\kappa^3}{\eta\sigma_u^2} \Leftrightarrow \\ \phi_\pi &= \left(\kappa\sigma + \frac{\sigma\kappa^3}{\eta}\right)\frac{\sigma_\varepsilon^2}{\sigma_u^2} + \frac{\kappa\sigma}{\eta} \Leftrightarrow \\ \phi_\pi^* &= \kappa\sigma\left(1 + \frac{\kappa^2}{\eta}\right)\frac{\sigma_\varepsilon^2}{\sigma_u^2} + \frac{\kappa\sigma}{\eta}. \end{aligned} \tag{11}$$

This is the expression for the optimal choice of ϕ_π presented in the exercise.

3) It is seen from (11) that the optimal value of ϕ_π depends negatively on η. The explanation is as follows. η measures the weight attached by the central bank to output gap fluctuations relative to inflation fluctuations. The higher is this parameter, the more costly are output gap fluctuations. We know that the presence of supply shocks (or cost-push shocks) in the model gives rise to a tradeoff for the central bank: When such a shock hits, the central bank must accept that it cannot fully stabilize both inflation and the output gap. If it wishes to reduce the deviation of one of these variables, it must accept a larger deviation of the other variable. When the central bank attaches a large weight to output stabilization, it is therefore willing to accept larger inflation fluctuations, and this is exactly what results in the case when the parameter η is relatively high, and the central bank's inflation response ϕ_π therefore is relatively low. A low value of ϕ_π implies a limited degree of inflation stabilization.

We can also observe directly that ϕ_π^* is increasing in the variance ratio, $\frac{\sigma_\varepsilon^2}{\sigma_u^2}$. In contrast to supply shocks, demand shocks do not cause a tradeoff for the central bank. After a demand shock, inflation and the output gap move in the same direction, so the central bank can stabilize both variables by setting a high value of ϕ_π. On the other hand, as discussed above, supply shocks force the central bank to choose between inflation stabilization and output gap stabilization. When demand shocks are relatively more important ($\frac{\sigma_\varepsilon^2}{\sigma_u^2}$ high), inflation and the output gap will move in the same direction more frequently, so the central bank will find it optimal to react strongly to inflation, thereby stabilizing both variables. On the other hand, when supply shocks dominate ($\frac{\sigma_\varepsilon^2}{\sigma_u^2}$ low), it will generally not be optimal to react strongly to inflation fluctuations, as this will lead to very large output fluctuations, as discussed above. In the limiting case when only demand shocks hit the economy ($\frac{\sigma_\varepsilon^2}{\sigma_u^2} \to \infty$), we see that $\phi_\pi^* \to \infty$, since the central bank can then eliminate any welfare loss, i.e., the "divine coincidence" applies.

4) Consider the solutions for x_t (9) and π_t (10) with no demand shocks:

$$x_t = -\frac{\phi_\pi}{\sigma + \phi_\pi \kappa} u_t,$$

$$\pi_t = \frac{\sigma}{\sigma + \phi_\pi \kappa} u_t.$$

The solution for π_t can be rewritten as:

$$\frac{\sigma + \phi_\pi \kappa}{\sigma} \pi_t = u_t,$$

which may then be inserted into the expression for x_t:

$$\begin{aligned} x_t &= -\frac{\phi_\pi}{\sigma + \phi_\pi \kappa} \frac{\sigma + \phi_\pi \kappa}{\sigma} \pi_t \Leftrightarrow \\ x_t &= -\frac{\phi_\pi}{\sigma} \pi_t. \end{aligned}$$

We can then insert the solution for ϕ_π^* found previously:

$$\begin{aligned} x_t &= -\frac{\kappa\sigma\left(1 - \frac{\kappa^2}{\eta}\right)\frac{\sigma_\varepsilon^2}{\sigma_u^2} + \frac{\kappa\sigma}{\eta}}{\sigma} \pi_t \Leftrightarrow \\ x_t &= -\left(\kappa\left(1 - \frac{\kappa^2}{\eta}\right)\frac{\sigma_\varepsilon^2}{\sigma_u^2} + \frac{\kappa}{\eta}\right) \pi_t. \end{aligned}$$

Finally, use the fact that there are no demand shocks (i.e., $\sigma_\varepsilon^2 = 0$), and rewrite:

$$x_t = -\left(0 + \frac{\kappa}{\eta}\right) \pi_t \Leftrightarrow$$

$$x_t = -\frac{\kappa}{\eta} \pi_t, \tag{12}$$

as desired. This expression is similar to the textbook optimality condition for optimal discretionary monetary policy (which is to be expected, since following a Taylor rule is equivalent to a non-commitment policy). The best students should point this out. The expression shows that if the rate of inflation increases due to a shock (in this case a cost-push shock, since it is assumed to be the only shock in the economy), the optimal policy involves a negative output gap, or "leaning against the wind". By pushing the output gap below zero, the central bank can dampen the inflationary pressure in the economy. More generally, the optimality condition again reflects the fact that in the face of supply shocks to the economy, the central bank faces a trade-off between stabilizing inflation and stabilizing the output gap.